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PREDICTION OF INCREMENTAL AIRCRAFT DRAG DUE TO EXTERNALLY CARRIED WEAPONS

by

David W. Lacey

July 1968

Report 2922
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LIST OF ILLUSTRATIONS

SYMBOLS

C_D	D/qS_w
C_{D_A}	aircraft drag coefficient
ΔC_D	$C_D - C_{D_A}$
c	local wing chord
d	maximum store diameter
l	store length
M	Mach number
q	free stream dynamic pressure, psf
S_m	store max cross sectional area $\pi d^2/4$
S_p	pylon planform area
S_t	store tail wetted area
S_w	aircraft wing area
ζ	lateral distance from store center line to aircraft center line
η	minimum depth
η_{max}	maximum depth, (see Figure 3)
κ	equivalent body length (see Figure 4)
λ	wing leading edge sweep angle
ξ	distance from store nose to wing leading edge
τ	equivalent body diameter (see Figure 4)

SUMMARY

The drag due to the external carriage of weapons on high speed attack aircraft results in large reduction in speed and range. Up to the present, the only means of evaluating this carriage drag has been either full scale flight test, or wind tunnel testing. Both methods are costly and time consuming. Accordingly, an analytical method is being sought which will provide useful estimates of the drag increment caused by external weapon installation without the need for such tests. The method, based on regression techniques, utilizes the large amount of wind tunnel data obtained from previous tests at NSRDC and other facilities. The prediction equation is determined as a function of the significant geometric parameters of the aircraft-store configuration. A highly encouraging degree of success has been achieved in application of the method to drag of stores mounted singly on pylons at Mach numbers of 0.8 and 0.9. The prediction equation has shown excellent agreement with drag data subsequently measured on arbitrary aircraft-store combinations. Effort to extend the scope of application is continuing.

INTRODUCTION

The drag caused by externally mounted weapons or Captive Flight drag has been a problem area in Aerodynamics for nearly as many years as man has been flying aircraft. During World War II, while there were performance losses due to external carriage of weapons, these losses were not sufficiently large to cause alarm. As speeds of aircraft increased, the problem became more critical. Whereas a World War II aircraft might suffer a reduction in speed of 20 miles an hour due to external fuel tanks, todays attack aircraft may lose 100 to 200 MPH. An example of this type of loss is shown in Figure 1. It might well be pointed out, that the situation depicted is not an extreme case and that there are many worse.

The initial investigations into the matter of captive flight drag at NSRDC were of limited scope. These investigations were, in general, merely determining experimentally the drag of a particular weapon carried on a particular aircraft. While these investigations presented useful data, they presented very little insight into the problems involved. It was not until 1960 that a systematic investigation was made into the overall problem of captive flight drag. Initial tests involved research models known as MIDS I, II, and III. The variables studied included wing sweep, store position, wing leading edge droop, nose shape, and others. At the same time that these studies were being conducted, additional testing involving current day Navy aircraft was initiated. Aircraft such as the A-4, A-7A, and F-8 were tested with many of the possible weapon loadings and the results published. Limited wind tunnel model investigations were also carried out on several Navy aircraft in order to reduce the drag of stores, and set up guidelines for future attack aircraft.

By 1966 sufficient experimental drag data had been accumulated from NSRDC, NASA, and other sources to warrant assembly into a convenient and usable form. Accordingly, data for aircraft with and without external weapons were recorded on punched cards and a computer program written. This program curve-fitted and solved for the incremental drag values, and automatically plotted the results to a suitable scale.

Very little attempt at correlation of these available data with significant parameters of the aircraft-store combination had been made in the past. It was decided, that by using the high speed digital computer and statistical techniques, correlation might successfully be attempted.

BACKGROUND FOR METHOD OF SOLUTION

It is well established from both experimental and theoretical aerodynamics that, for practical engineering purposes

$$C_D = C_D(M_\infty , Re, \text{Body Geometry})$$

Within the ranges of variables of greatest immediate interest, differences in incremental drag due to Reynolds number are usually of secondary importance, and it was decided to neglect these differences.

The key to successful correlation is to describe the body geometry in terms of a manageable number of the most important nondimensional geometric parameters. These might include nondimensional combinations of length, cross sectional area, diameter, etc. The drag coefficient at a given Mach number might then be written as

$$C_D \cong G\left(x_1, x_2, \ldots, x_n\right)$$

where $x_1, \ldots, x_n$ are geometric parameters.

If C_D is thought to be a linear expansion of the parameters, then

$$C_D \cong a_o + a_1 x_1 + a_2 x_2 + \ldots a_n x_n$$

Additionally it might be necessary to use higher order powers of the parameters and/or cross product terms, so that

$$C_D \cong a_o + a_1 x_1 + , \ldots + a_n x_n + b_1 x_1^2, + \ldots, + b_n x_n^2$$

$$+ c_{12} x_1 x_2 + \ldots + c_{n-1, n} x_{n-1} x_n + \ldots$$

For the $\underline{k}$th body of a set of many bodies, the equation becomes

$$C_{D_k} \cong a_o + a_i x_{ik} + b_i x_{ik}^2 + c_{ij} x_{ik} x_{jk} + \ldots$$

If sufficient experimental data are available the unknown coefficients a_o, a_i, b_i, c_{ij} may be found by statistical methods.

MULTIPLE LINEAR REGRESSION

The statistical method chosen to solve for the unknown coefficients is multiple linear regression. Multiple linear regression involves the fitting of a surface in a "K" dimensional space. This surface is the best possible fit of an assumed value such that, its deviation from the true value is minimized. The form of the regression equation is

$$\alpha_k' = a_o + a_1 \beta_{1k} + a_2 \beta_{2k} + \cdot \cdot \cdot + a_n \beta_{nk}$$

where α_K' is the estimated value of α_k and the β_{ik}, $i = 1, n$ are the known parameters.

Measuring the parameters from their means, the equation may be rewritten as

$$\alpha_k'' = a_1'' \beta_{1k}'' + a_2'' \beta_{2k}'' + \cdot \cdot \cdot + a_n'' \beta_{nk}''$$

where $\alpha_k'' = \alpha_k' - \bar{\alpha}$

and $\beta_{ik}'' = \beta_{ik}' - \bar{\beta}_i, \; a_i'' = a_i$

and $a_o = a_1 \bar{\beta}_1 + \cdot \cdot \cdot + a_n \bar{\beta}_n - \bar{\alpha}'$

it can be shown (see the appendix) that, from the above, a system of equations may be generated with respect to the least squares criterion, and the unknown a_i's may be found. The regression equation has the same form as the equation for C_D if:

$$\alpha_k' = C_{D_K}$$

$$a_o = a_o, \; a_i = a_i, \; b_i \text{ or } c_{ij}$$

and $\beta_{ik} = x_{ik}, \; x_{ik}^2 \text{ or } x_{ik} x_{jk}$

In addition to the solution of the unknown a_i's, two additional coefficients may be obtained from the regression equation. The first is called the multi-correlation coefficient. This coefficient gives an indication of the goodness of fit of the regression equation. The second coefficient is called the partial correlation coefficient. This indicates the importance of a single parameter in the overall equation. The mathematical definition of these coefficients is given in the appendix.

PARAMETERS

The parameters which are used to describe the geometry of the wing pylon-store configuration include store position relative to the wing, reference areas of store and pylon, store fineness ratio and wing leading edge sweep angle. In addition, at certain Mach numbers, area rule effects become important as well as drag interference between store upper surface and wing lower surface.

With the inclusion of an area rule parameter and interference parameters the final correlation equation for application to data at a single Mach number is:

$$\Delta C_D = a_o + a_1 \left(\eta/\eta_{max}\right) + a_2 (\eta/d) + a_3 (\xi/c) + a_4 (\zeta/d)$$

$$+ a_5 \left(S_p/S_w\right) + a_6 (l/d) + a_7 \left(S_t/S_m\right) + a_8 \left(S_m/S_w\right)$$

$$+ a_9 (\cos\lambda) + a_{10} ((\tau/\varkappa)\cos\lambda) + a_{11} \left(\eta/\eta_{max}\right)^2$$

$$+ a_{12} \left(S_m/S_w\right)^2 + a_{13} (\cos\lambda)^2 + a_{14} ((\tau/\varkappa)\cos\lambda)^2 + a_{15} (\eta/d)^2$$

$$+ a_{16} \left(\eta/\eta_{max}\right) (\eta/d) + a_{17} \left(\eta/\eta_{max}\right) (\xi/c)$$

The various parameters are shown in Figure 2.

The interference parameter $\left(\eta/\eta_{max}\right)$ is shown in Figure 3. The ratio approximates the familiar A/A* of Laval nozzle theory. When the ratio becomes small, the chance of sonic flow becomes large and therefore, large interference may occur.

Limited area rule has been taken into account by the use of the parameter $((\tau/\varkappa)\cos\lambda)$, and is shown in Figure 4. The ratio $(\tau/\varkappa)^{-1}$ is the fineness ratio of an equivalent body encompassing part of the wing, pylon, and body as shown in Figure 4.

Using the correlation equation, good agreement between predicted and experimental data has been obtained. This is shown in Figure 5 for Mach numbers of 0.8 and 0.9. Figure 5 presents data for forty-nine aircraft store configurations consisting of eleven aircraft and fourteen stores. The data are presented in the form of C_D and are based on an accuracy of ± 0.0010.

ASSESSMENT OF PARAMETERS

The importance of various parameters on incremental drag may best be assessed by use of the aforementioned partial correlation coefficient. The effect of a change in incremental drag caused by a change in a parameter is determined by the sign of its coefficient in the regression equation. Utilizing for each parameter, the partial correlation coefficient and the regression coefficient associated with the parameter, the following were observed:

1. The interference parameters $\left(\eta/\eta_{max}\right)$ and $\left(\eta/\eta_{max}\right)^2$ had little effect at $M = 0.8$. At $M = 0.9$ interference became important and incremental drag decreased with an increase in $\left(\eta/\eta_{max}\right)$. In general $\left(\eta/\eta_{max}\right)$ is less than 1 and the ratio should be as close to 1 as possible for low incremental drag at high subsonic Mach numbers.

2. The depth beneath the wing (η/d) and $(\eta/d)^2$ were found to be of equal importance at both Mach numbers evaluated. An increase in (η/d) produced a corresponding increase in incremental drag.

3. The longitudinal position parameter (ξ/c) was important at both Mach numbers. The incremental drag increased with an increase in (ξ/c).

4. Variation of incremental drag with lateral position (ζ/d) was small. An increase in (ζ/d) produced a small decrease in incremental drag.

5. Pylon size in relation to wing area $\left(S_p/S_w\right)$ was important at $M = 0.8$. Where an increase in $\left(S_p/S_w\right)$ produced a larger incremental drag. At $M = 0.9$ the importance of the parameter was not as noticeable and the effect on drag was reversed.

6. The size of the stores tail surfaces in relation to cross sectional area $\left(S_t/S_m\right)$ was important at both Mach numbers. The effect on drag was constant at both $M = 0.8$ and $M = 0.9$. As the ratio increased, the incremental drag increased.

7. The effect on drag caused by a change in fineness ratio (l/d) was important at both $M = 0.8$ and $M = 0.9$. As expected an increase in fineness ratio produced a decrease in incremental drag.

8. The size of a store carried on a particular aircraft $\left(S_m/S_w\right)$, and $\left(S_m/S_w\right)^2$ was of relatively minor importance at $M = 0.8$. At $M = 0.9$, however, the effect was more pronounced. An increase in $\left(S_m/S_w\right)$ produced a corresponding increase in incremental drag.

9. Wing leading edge sweep angle $(\cos \lambda)$ and $(\cos \lambda)^2$ was of relative importance at both Mach numbers. At $M = 0.8$, an increase in $\cos \lambda$, produced a decrease in incremental drag. At $M = 0.9$ the effect on drag caused by an increase in $(\cos \lambda)$ was reversed.

10. At $M = 0.8$ the limited area rule parameters $((\tau/\kappa)\cos \lambda)$ and $((\tau/\kappa)\cos \lambda)^2$ have little effect. At a Mach number of 0.9 the parameter was of large importance. An increase in the parameter produced an increase in drag.

11. The cross product term $\left(\eta/\eta_{max}\right)(\eta/d)$ was of extreme importance at both Mach numbers. The effect on drag was opposite to that produced by (η/d) and $(\eta/d)^2$. An increase in the cross product parameter produced a decrease in incremental drag at both Mach numbers.

12. The other cross product parameter $\left(\eta/\eta_{max}\right)$ $(\xi/c$ was, likewise, of large importance at both Mach numbers. An increase in the parameter produced a decrease in incremental drag.

FUTURE PLANS

At present the prediction works well for singly mounted bodies of revolution in the transonic speed range. Attack aircraft, however, are not limited to a single carriage per wing, but often have provisions for multiple carriage of stores. Many of the stores carried are not plain bodies of revolution but have complex shapes, such as multiple bomb racks and winged missiles. Plans are being made to incorporate the effects of these complex configurations in the prediction equation. Multiple bomb racks may require the use of a fictitious body having the same drag characteristics as the rack, as well as additional parameters for proper drag determination. The problem of side-by-side store mounting will undoubtedly require additional terms in the equation in order to account for store-to-store interference.

Optimization of store position for minimum drag will also be attempted. Wind tunnel tests at Naval Ship Research and Development Center indicated that significant gains in performance can be attained, by proper store positioning. Use of the correlation equation for purposes of optimization is very promising, and does not require the time and expense associated with wind tunnel testing.

As more data become available the equation will be further refined and its speed range extended to low supersonic speeds.

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July 1968

APPENDIX

MULTIPLE LINEAR REGRESSION

Given n sets of data in terms of k parameters, suppose that a dependent variable or yield parameter, X_1, may be approximated by a linear combination of the other k - 1 parameters. Then a general equation would be of the form

$$X_1' = B_o + B_2 X_2 + \ldots B_k X_k$$

where X_1' is the estimated value of X_1, and X_j, $j = 2 \ldots k$ are the k - 1 parameters.

Measuring the parameters from their means, the equation may be rewritten

$$x_1' = b_o + b_2 x_2 + \ldots b_k x_k$$

where

$$x_i = X_i - \bar{X}_i, \ i = 1, \ldots k$$

$$b_o = B_2 \bar{X}_2 + \ldots B_k \bar{X}_k + B_o - \bar{X}_1$$

$$b_i = B_i, \ i = 2, \ldots k$$

For X_1' to be the best linear estimate of X_1, then with respect to the least squares criterion, $\sum (X_1 - X_1')^2$ must be minimized. This is equivalent to minimizing $\sum (x_1 - x_1')^2$; i.e.,

$$\sum (x_1 - b_o - b_2 x_2 - \ldots b_k x_k)^2$$

A necessary and sufficient condition for minimization of this is

$$\sum_{\substack{j=c \\ j \neq 1}}^{k} \sum^{n} \frac{\partial}{\partial b_j} (x_1 - b_o - b_2 x_2 - \ldots b_k x_k)^2 = 0$$

writing out the k equations gives

$$b_o n + b_2 \sum x_2 + \ldots b_k \sum x_k = \sum x_1$$

$$b_o \sum x_2 + b_2 \sum x_2^2 + \ldots b_k \sum x_2 x_k = \sum x_1 x_2$$

$$\vdots$$

$$b_o \sum x_k + b_2 \sum x_2 x_k + \ldots b_k \sum x_k^2 = \sum x_1 x_k$$

Note that $\sum x_i = \sum (X_i - \bar{X}_i) = 0$. Thus each term in the first equation vanishes, and $b_o = 0$. The system becomes

$$b_2 \sum x_2^2 + \ldots + b_k \sum x_k x_2 = \sum x_1 x_2$$

$$b_2 \sum x_2 x_3 + \ldots + b_k \sum x_k x_3 = \sum x_1 x_3$$

$$\vdots$$

$$b_2 \sum x_2 x_k + \ldots + b_k \sum x_2^2 = \sum x_1 x_k$$

These are the normal equations, solvable for b_j, $i = 2, \ldots k$.

The coefficients of the b's can be expressed in the following statistical quantities:

$$\text{Let } s_i = \sqrt{\frac{1}{n-1} \sum^{n} (X_i - \bar{X}_i)^2}$$

= sample standard deviation of X_i

$$r_{ij} = \frac{\sum x_i x_j}{n\, s_i\, s_j}$$

= sample correlation coefficient between X_i and X_j

Then

$$\sum x_i x_j = n\, r_{ij}\, s_i\, s_j$$

Cancelling common factors after these substitutions have been made, the normal equations become

$$
\begin{aligned}
b_2 r_{22} s_2 + b_3 r_{23} s_3 + \ldots b_k r_{2k} s_k &= r_{21} s_1 \\
b_2 r_{32} s_2 + b_3 r_{33} s_3 + \ldots b_k r_{3k} s_k &= r_{31} s_1 \\
&\vdots \\
b_2 r_{k2} s_2 + b_3 r_{k3} s_3 + \ldots b_k r_{kk} s_k &= r_{k1} s_1
\end{aligned}
$$

Assuming that the determinant of the coefficients is not zero, then

$$
b_i = \frac{s_1 s_2 \cdots s_{i-1} s_{i+1} \cdots s_k}{s_2 s_3 \cdots s_k}
\frac{\begin{vmatrix} r_{22} & r_{23} & \cdots & r_{21} & r_{2k} \\ r_{32} & r_{33} & \cdots & r_{31} & r_{3k} \\ & \vdots & & & \\ r_{k2} & r_{k3} & \cdots & r_{k1} & r_{kk} \end{vmatrix}}{\begin{vmatrix} r_{22} & r_{23} & \cdots & r_{2i} & r_{2k} \\ r_{32} & r_{33} & \cdots & r_{3i} & r_{3k} \\ & \vdots & & & \\ r_{k2} & r_{k3} & \cdots & r_{ki} & r_{kk} \end{vmatrix}}
$$

Note that the determinants in the numerator and denominator differ only in the elements occurring in the i - 1st column. Consider the following determinant:

$$
R = \begin{vmatrix} r_{11} & r_{12} & r_{13} & \cdots & r_{1k} \\ r_{21} & r_{22} & r_{23} & \cdots & r_{2k} \\ & \vdots & & & \\ r_{k1} & r_{k2} & r_{k3} & \cdots & r_{kk} \end{vmatrix}
$$

The determinant in the denominator of b_i is the minor of r_{11} in R, denoted R_{11}. Also, the numerator determinant of b_i can be converted to the minor of r_{1i} in R by shifting the column headed by r_{21} to the first column position. Using the relation between minors and cofactors, the numerator determinant becomes $- R_{1i}$. Thus

$$b_i = \frac{-s_1 R_{1i}}{s_i R_{11}}$$

The usefulness of the first equation for estimation, i.e.,

$$X_1' = Bo + B_2 X_2 + \ldots + B_k X_k$$

is indicated by its multiple correlation coefficient defined in Reference 1, as

$$r_{1.23\ldots k} = \sqrt{1 - \frac{R}{R_{11}}}$$

If $r_{1.23\ldots k}$ is close to 1, this indicates that the data points lie near the derived "plane," while if $r_{1.23\ldots k}$ is close to zero then either the relationship is weakly linear, or curvilinear.

Similarly, the influence of a particular parameter is reflected in its partial correlation coefficient:

$$r_{1j.23\ldots k} = \frac{-R_{1j}}{\sqrt{R_{11} R_{jj}}}$$

REFERENCE

1. Hoel, Paul G. Introduction to Mathematical Statistics. N. Y., Wiley [1947] 258 p. illus.

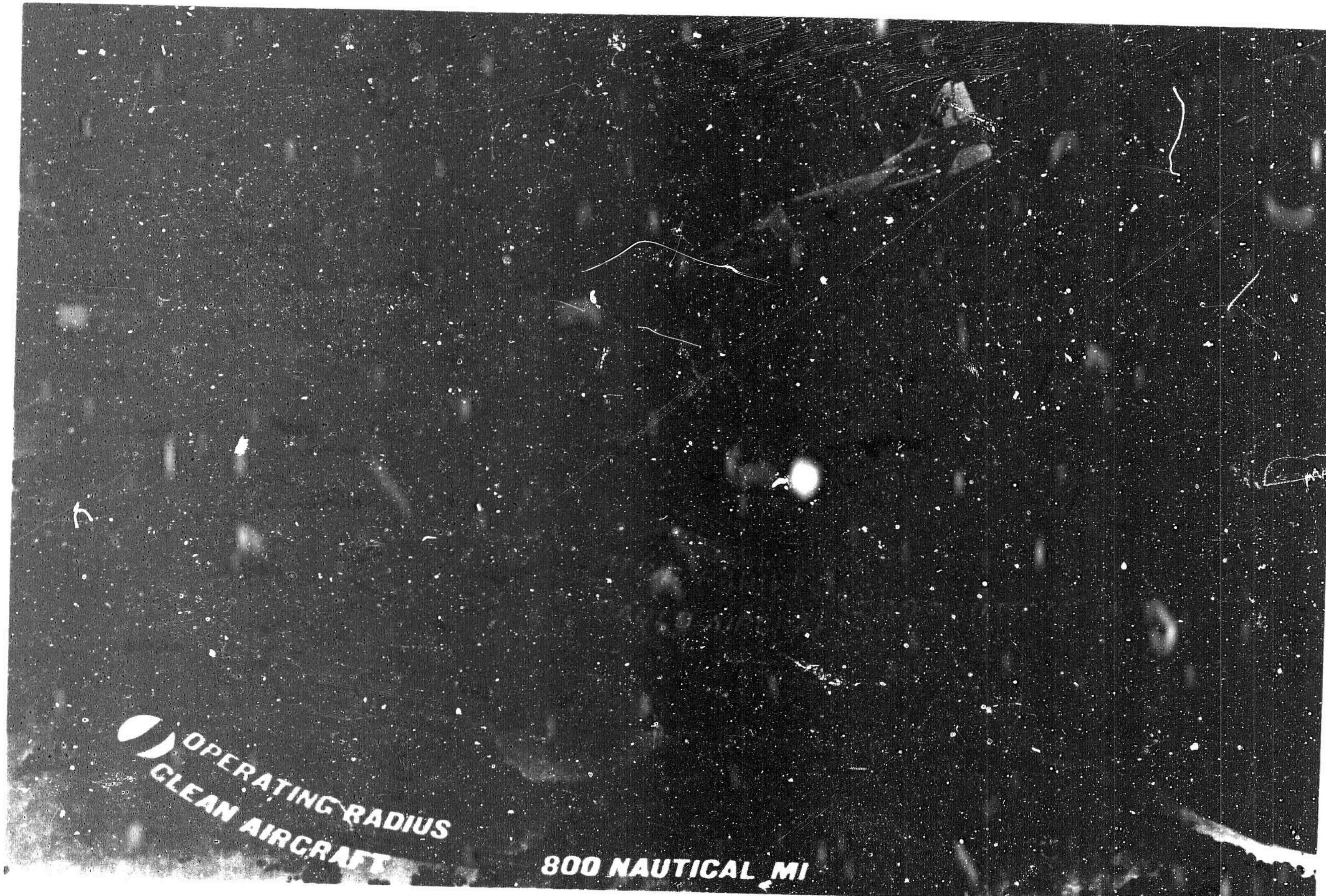

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Figure 1 - Effect of Stores on Aircraft Range

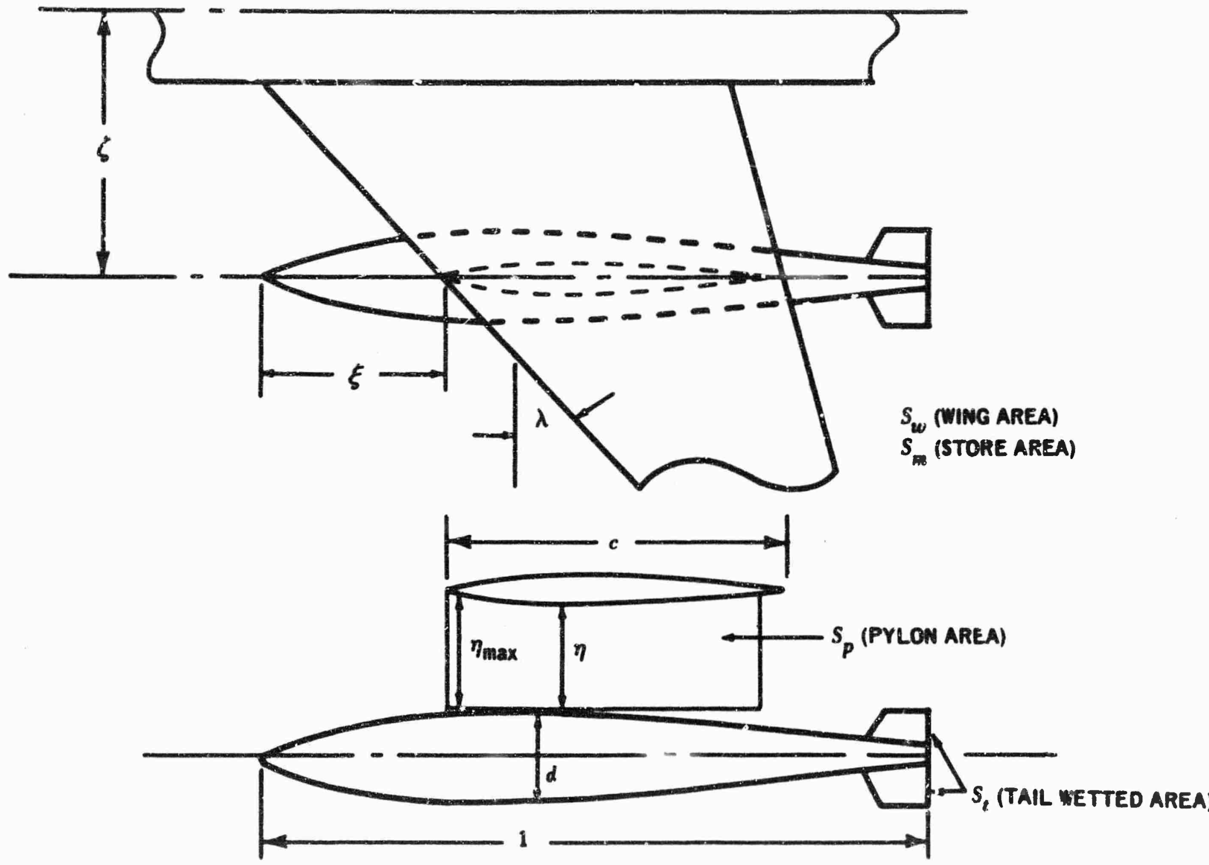

Figure 2 – Parameter Definition

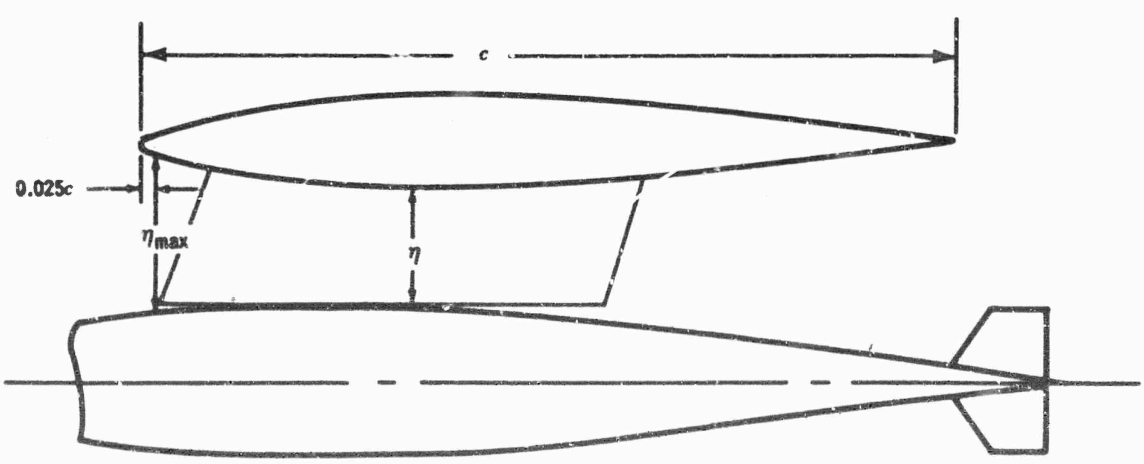

Figure 3 – Definition of Interference Parameter (η/η_{max})

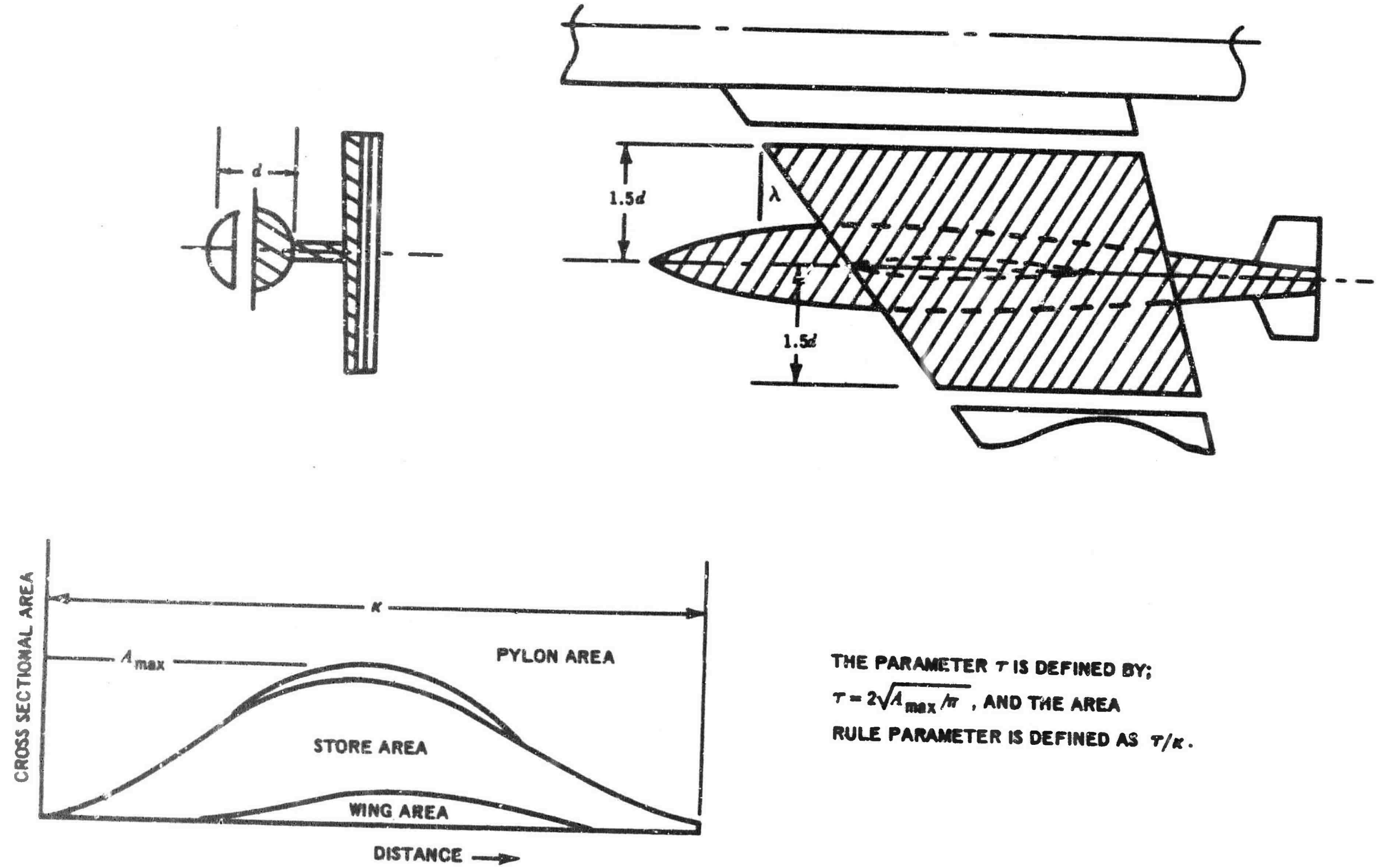

Figure 4 – Definition of Area Rule Parameter

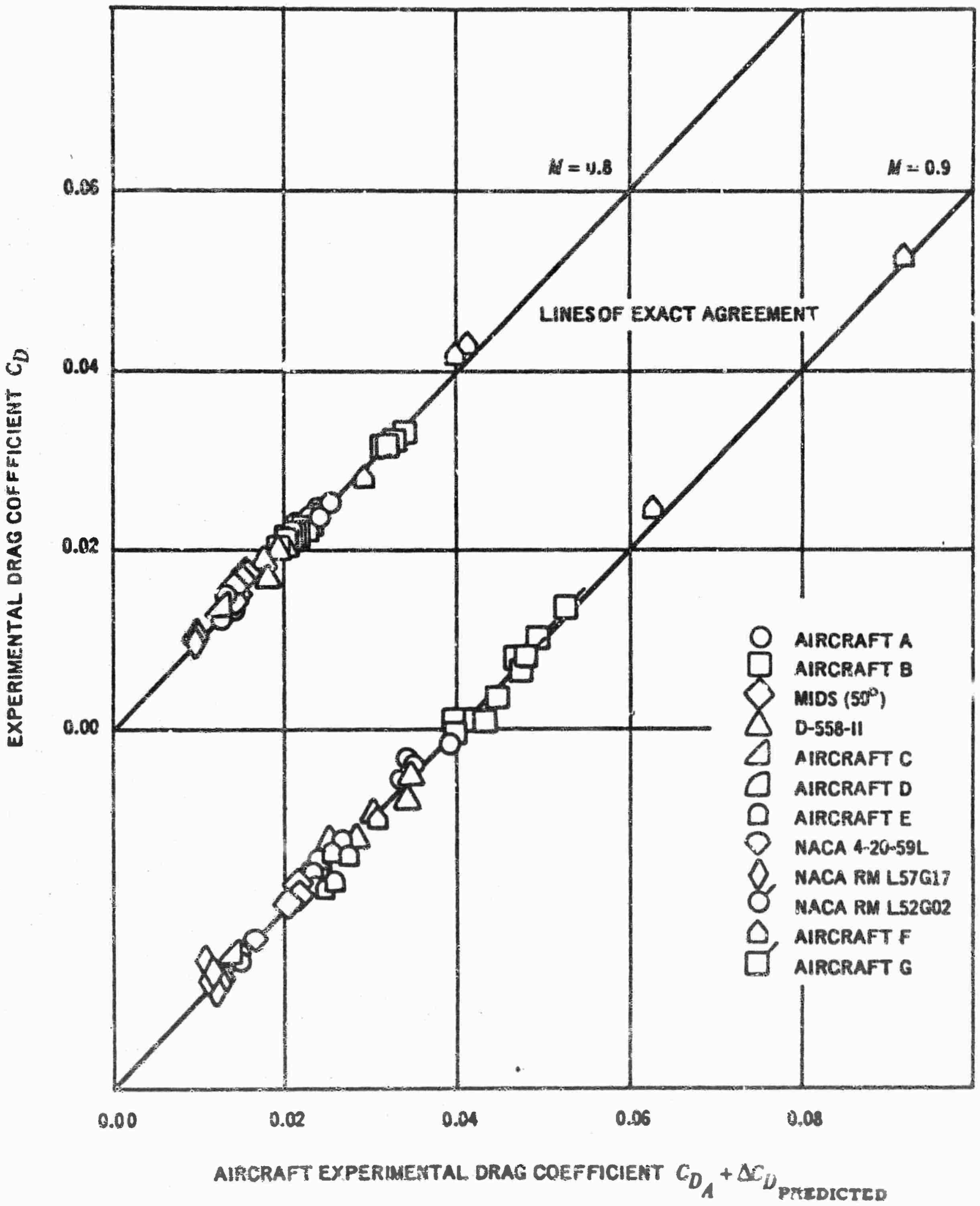

Figure 5 - Results of Prediction Equation for 49 Configurations at Mach Numbers of 0.8 and 0.9

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13. ABSTRACT

The drag due to the external carriage of weapons on high speed attack aircraft results in large reduction in speed and range. Up to the present, the only means of evaluating this carriage drag has been either full scale flight test, or wind tunnel testing. Both methods are costly and time consuming. Accordingly, an analytical method is being sought which will provide useful estimates of the drag increment caused by external weapon installation without the need for such tests. The method, based on regression techniques, utilizes the large amount of wind tunnel data obtained from previous tests ~~at NSRDC and other facilities~~. The prediction equation is determined as a function of the significant geometric parameters of the aircraft-store configuration. A highly encouraging degree of success has been achieved in application of the method to drag of stores mounted singly on pylons at Mach numbers of 0.8 and 0.9. The prediction equation has shown excellent agreement with drag data subsequently measured on arbitrary aircraft-store combinations. Effort to extend the scope of application is continuing.

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